Exercise 31: Walking the grid

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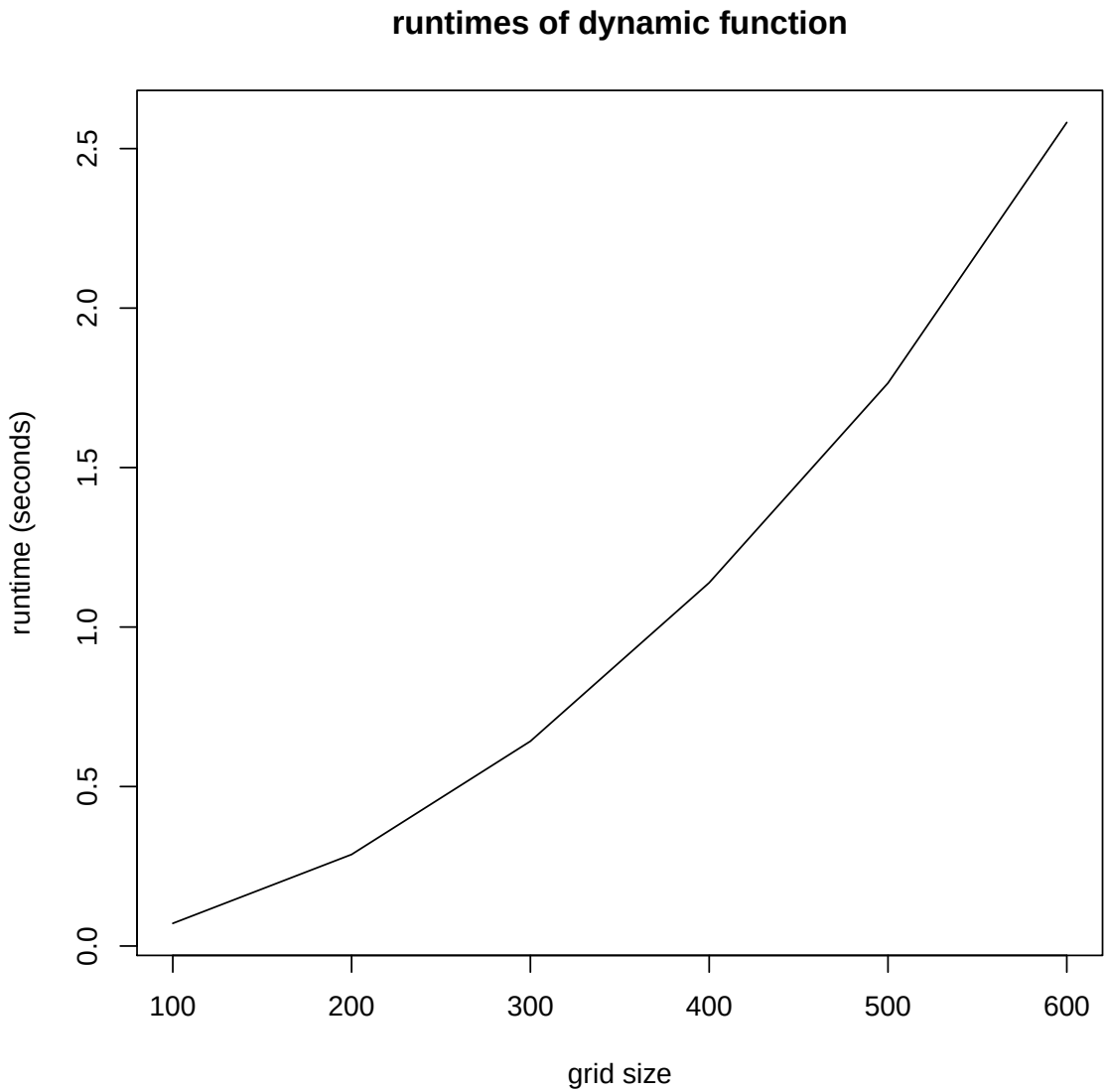
Dynamic Programming

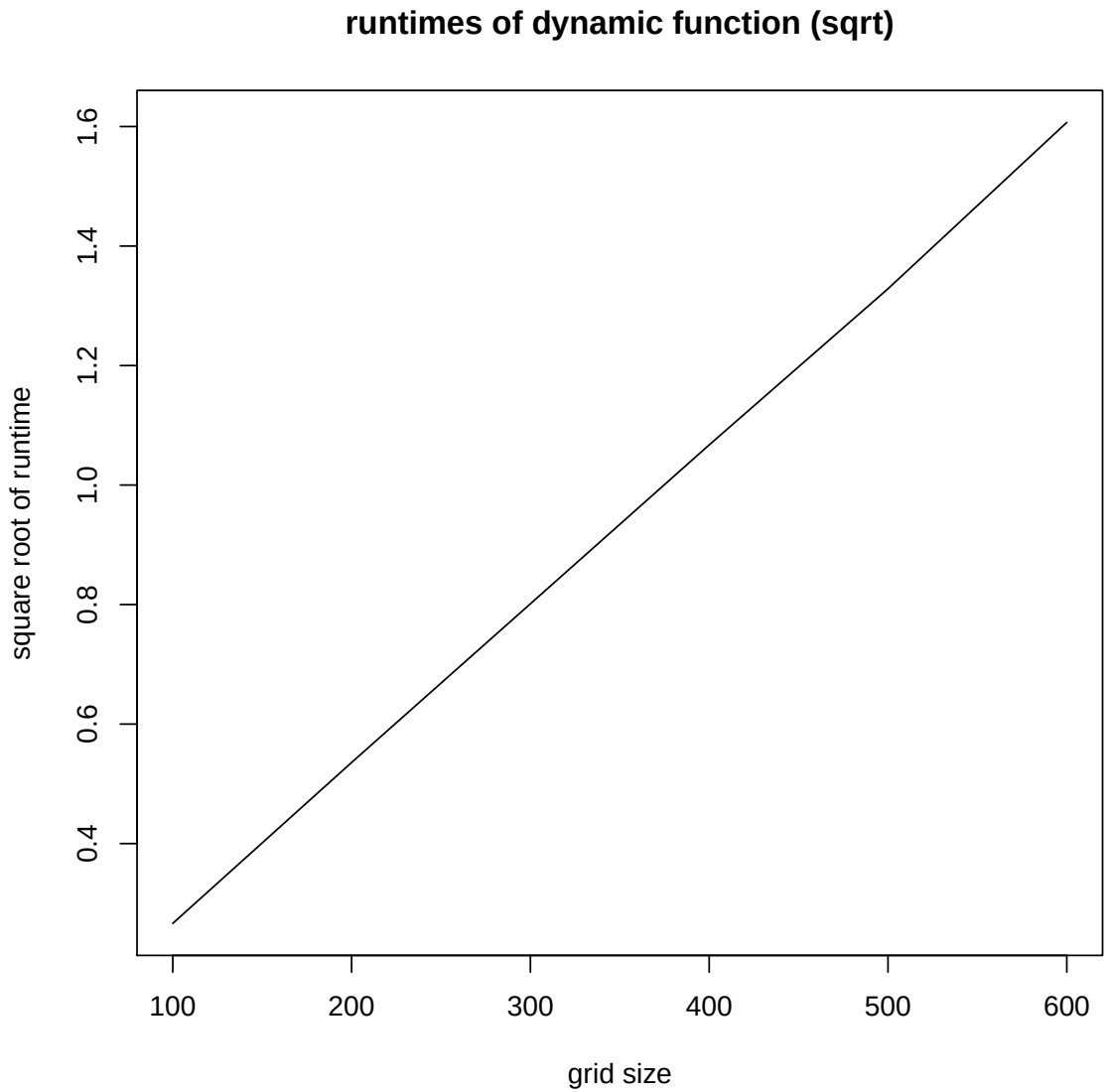
in R code

```
number_of_grid_routes_dynamic <- function(n)
{
  m <- matrix(0, n+1, n+1)
  m[n+1,] <- 1
  m[,n+1] <- 1
  for(x in n:1)
    for(y in n:1)
      m[x,y] <- m[x+1,y] + m[x,y+1]
  return(m[1,1])
}
```

Expected runtime

Note the **two nested for loops** (with **n iterations** each) $\Rightarrow$ the runtime will be $O(n^2)$


runtimes of dynamic function
runtime (seconds)
0.0
0.5
1.0
1.5
2.0
2.5
100
200
300
400
500
600
grid size


runtimes of dynamic function (sqrt)
square root of runtime
0.4
0.6
0.8
1.0
1.2
1.4
1.6
100
200
300
400
500
600
grid size

grid size	100	200	300	400	500	600
runtime	0.071	0.287	0.642	1.139	1.765	2.582
$\sqrt{\text{runtime}}$	0.266	0.536	0.801	1.067	1.329	1.607
$\frac{\sqrt{\text{runtime}}}{\text{grid size}}$	0.00266	0.00268	0.00267	0.00267	0.00266	0.00268

mean of $\frac{\sqrt{\text{runtime}}}{\text{grid size}}$: 0.00267

estimated runtime: $(0.00267 \cdot n)^2$

How long would it take to find the number of routes through an 80×80 grid?

$(0.00267 \cdot 80)^2 = 0.0456$ (seconds)

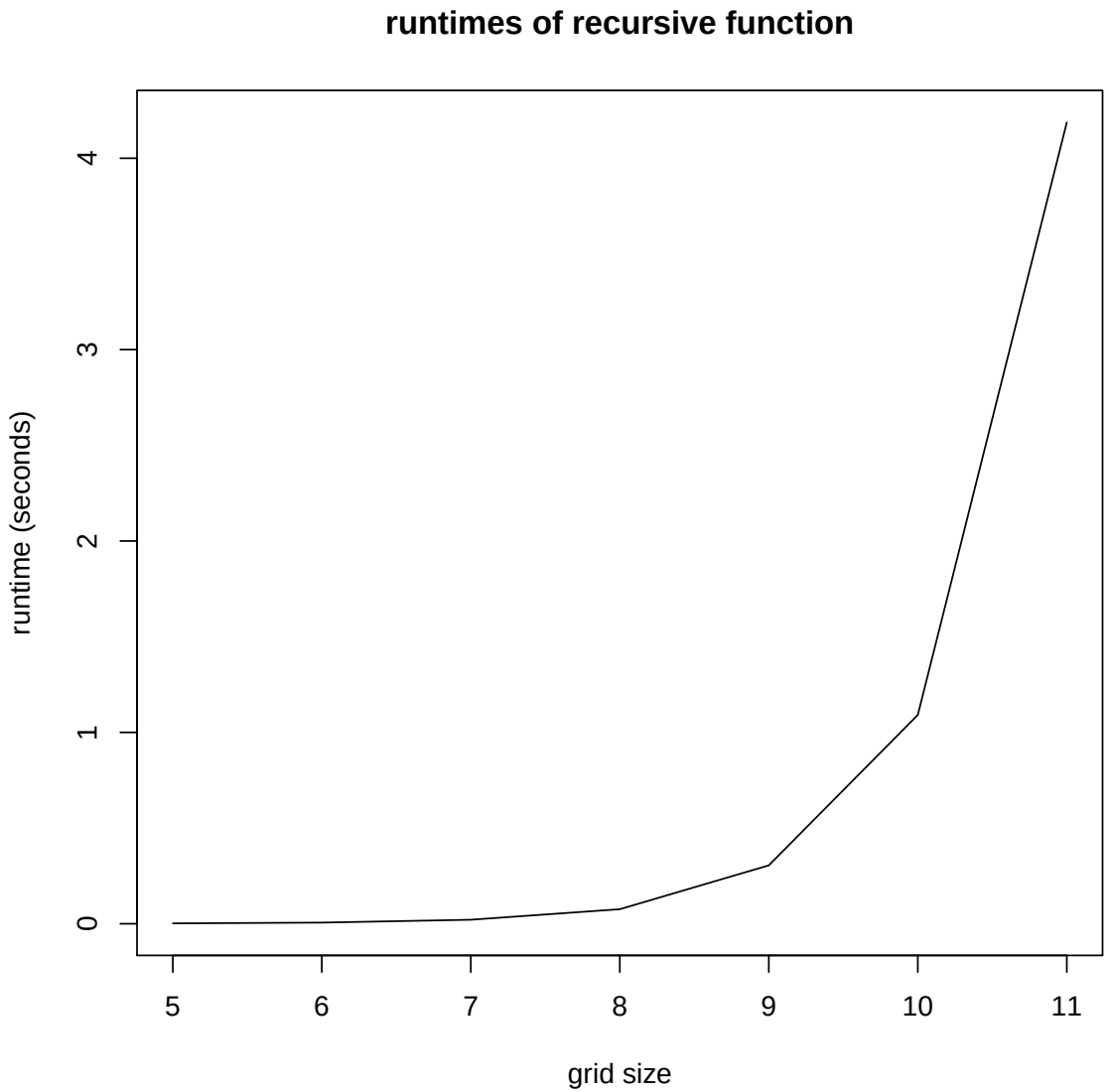
Recursion

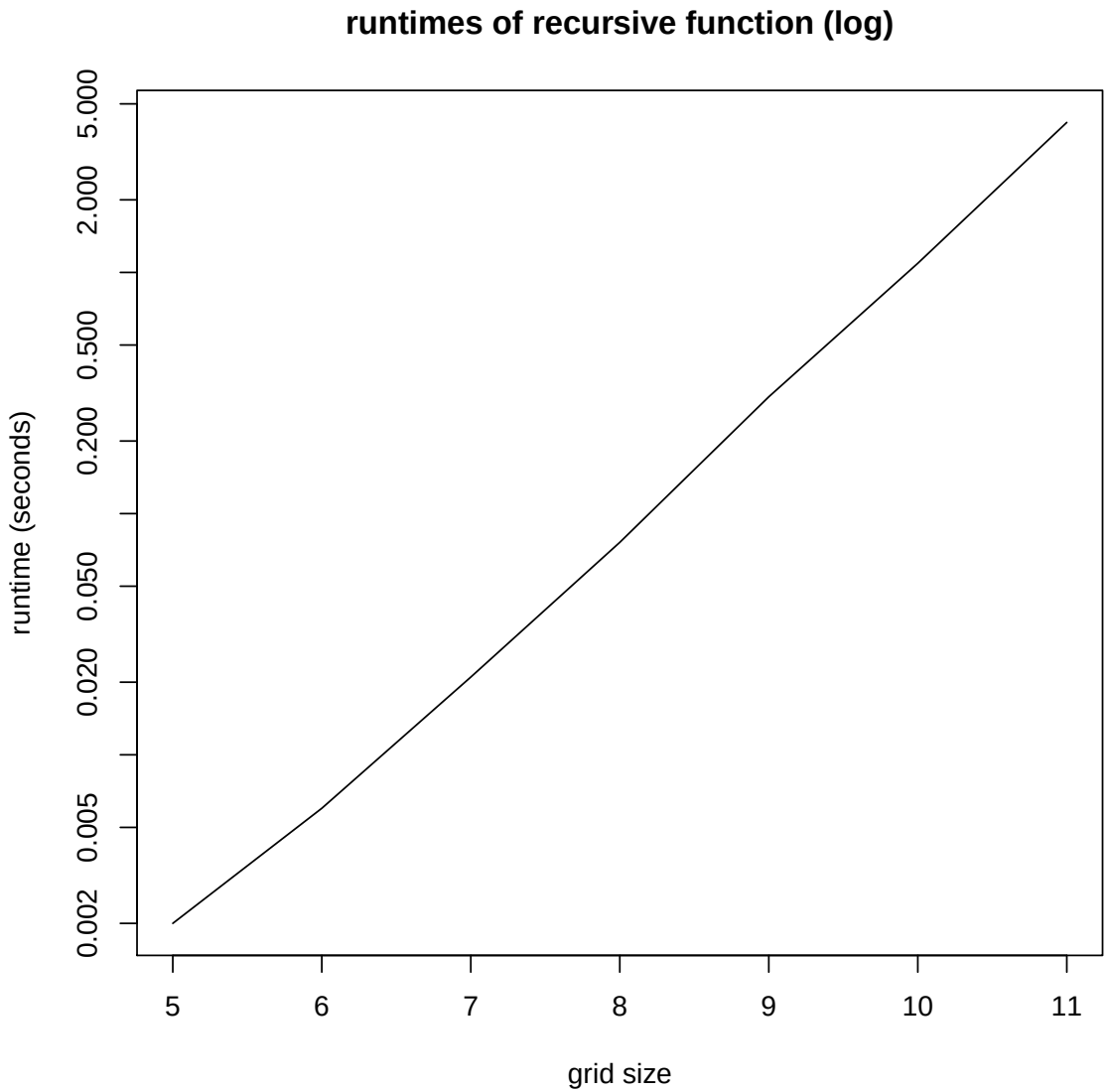
in R code

```
number_of_grid_routes_recursive <- function(x, y=x)
{
   if(!x*y) return(1)
   return(number_of_grid_routes_recursive(x-1,y) +
        number_of_grid_routes_recursive(x,y-1))
}
```

Expected runtime

Note that due to the **recursion** the count of function calls **rises exponentially** $\Rightarrow$ the runtime will be $O(2^n)$


runtimes of recursive function
runtime (seconds)
0
1
2
3
4
5
6
7
8
9
10
11
grid size


runtimes of recursive function (log)
runtime (seconds)
0.002
0.005
0.020
0.050
0.200
0.500
2.000
5.000
5
6
7
8
9
10
11
grid size

grid size	5	6	7	8	9	10
runtime	0.002	0.006	0.021	0.076	0.305	1.091
log_2(runtime)	-8.97	-7.38	-5.57	-3.72	-1.71	0.13
diff (log_2(runtime))	1.59	1.81	1.85	2.01	1.84	1.94

mean of *diff* (log_2(runtime)): 1.84 (= slope)

intercept: $-8.97 - 5 \cdot 1.84 = -18.17$

estimated runtime: $2^{1.84 \cdot n - 18.17}$

How long would it take to find the number of routes through an 80×80 grid?

$2^{1.84 \cdot 80 - 18.17}$ s $\approx 2^{129}$ s $\approx 2.25 \cdot 10^{31}$ years $\approx 1.6 \cdot 10^{21}$ universe age